IPM component 3

Term 1, week 3

On absolutism and relativism

Introduction

- Here we address the aspect of whether there are such things as
  - absolute facts: things that are absolutely true and will never change,

  or
  - relative facts: things which are true only in certain circumstances but not true in other circumstances.

Exercise

1) Is the arithmetic below correct? If so, why? If not why not?

1 + 0 = 0	1 + 1 = 10				
1 + 0 = 0	1 + 1 = 2	1 + 2 = 10			
1 + 0 = 0	1 + 1 = 2	1 + 2 = 3	1 + 3 = 10		
1 + 0 = 0	1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 10	
….	….	….	…	…	…

Exercise

2) Do all angles in a triangle always add up to 180^0 ?

3) are two lines that are initially paralle always parallel?

Examples

1) *The square root of 2*

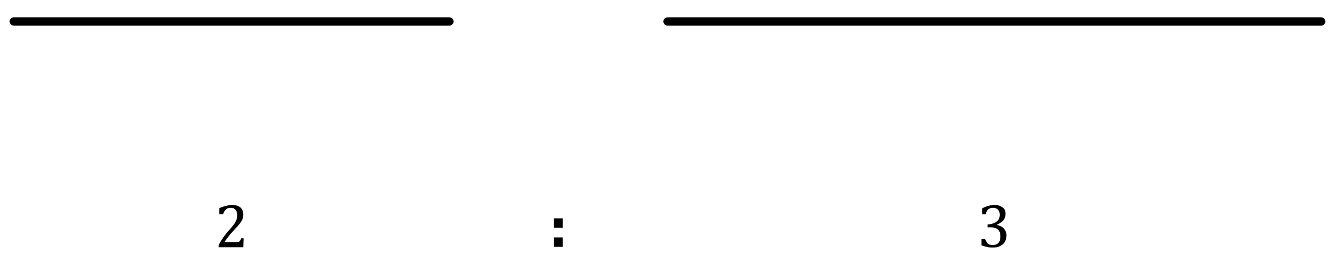
- Ancient Greek mathematics consisted of the study of numbers and geometry.

Numbers	Geometry
Only whole numbers 1, 2, 3, 4,	The relationships between lines, curves, angles, planes, etc.

Examples

1) *The square root of 2*

- They associated numbers to lines. So, "1" was associated with a line of a given length. "2" was associated with a line twice that length, etc.



Examples

1) *The square root of 2*

- Because numbers were associated with physical measurements, the Greeks did not believe negative numbers existed (nothing has a negative length, area, weight, etc.)
- They did not have fractions. A line was a line, whatever its length.

Examples

1) *The square root of 2*

- There was no such thing as half a line since this was also a line. So they only thought in wholes.

- But they did have ratios. As such the length (an integer) of one line was compared with the length (another integer) of another line.

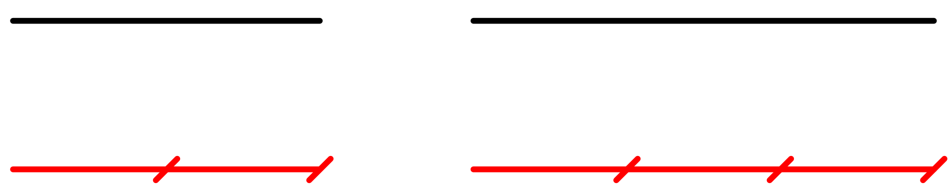
Examples

1) *<u>The square root of 2</u>*

- So numbers were used as shorthand for comparing lengths of lines.

Two lines

Confirmation of co-measurability



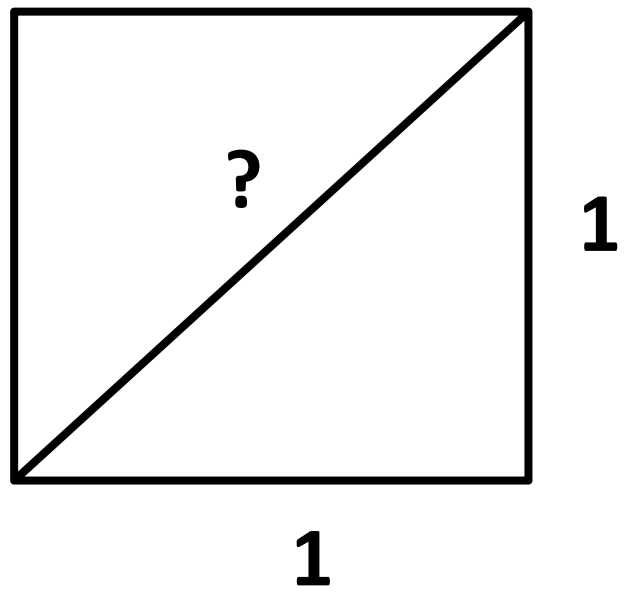
Ratio: Using numbers as a short-hand for lines of different magnitudes.

2 : 3

Examples

1) *The square root of 2*

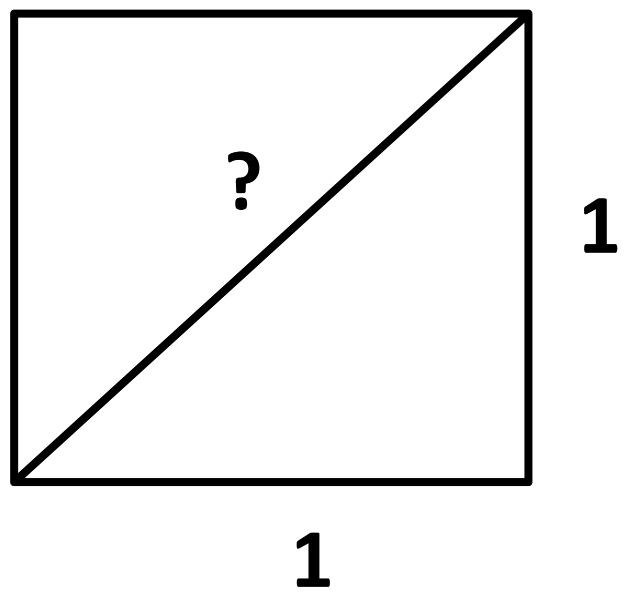
- Can we do the same integer comparison with a unit square?
- Can we compare the sides of square with its diagonal?



Examples

1) *The square root of 2*

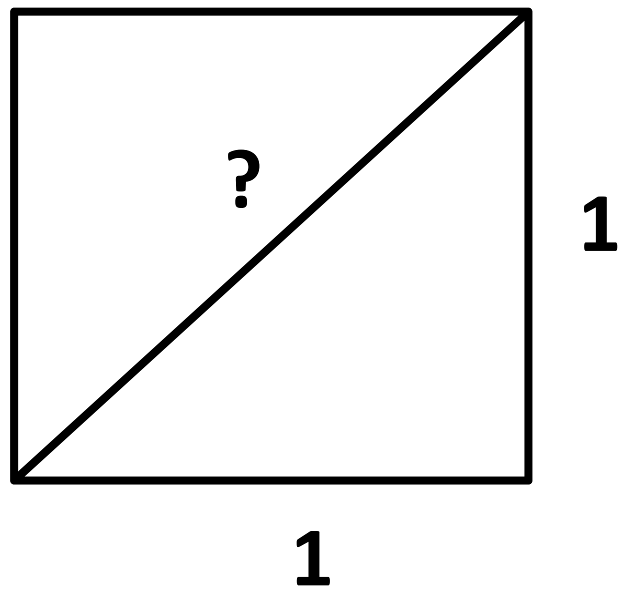
- Can we do the same integer comparison with a unit square?
- Can we compare the sides of square with its diagonal?
- **No.** There is no line which acts as a common measure



Examples

1) *The square root of 2*

- The length of the diagonal is what we now call $\sqrt{2}$
- This cannot be expressed as *a* : *b* where *a* and *b* are integers.



Examples

1) *The square root of 2*

- This implies two things:
  - The diagonal exists as a geometric object, i.e. it exists as a line,
  - No number can be associated with this line, hence $\sqrt{2}$ does not exist.

Examples

- Other examples of things which cannot exists:
  - Negative numbers: there is no such thing as negative length, weight, pressure, etc.
  - Imaginary numbers: There is no solution to $x^2 = -1$ since the square of any number is always positive.
- Such has been the opinion of experts over history.

Examples

2) *Infinity and the infinitely small*

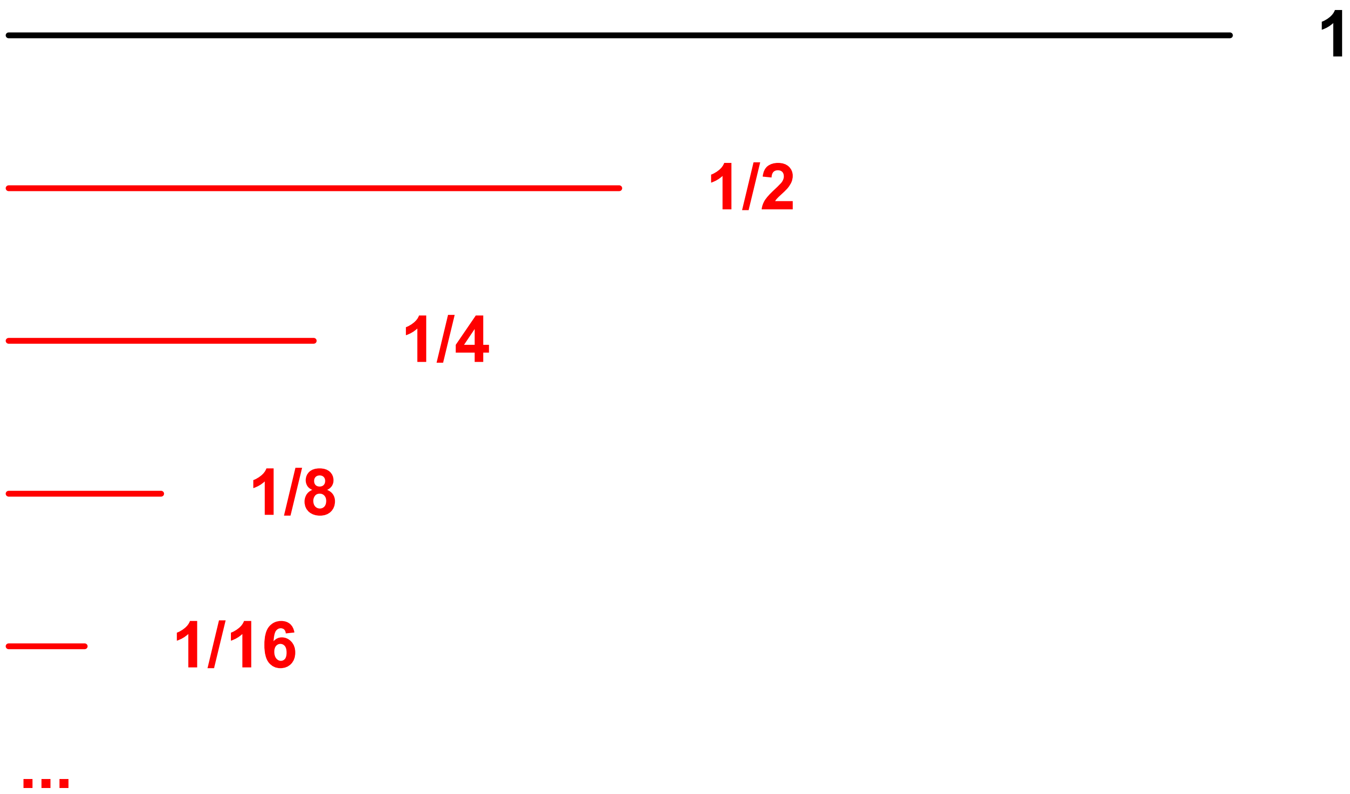
- Consider a string of length 1. This clearly has a finite length:

1

- Now cut it in half, and cut the half in half, and cut that half in half. Repeat forever.

Examples

2) *Infinity and the infinitely small*



Examples

2) *Infinity and the infinitely small*

- Mathematically this gives

$$1/2 + 1/4 + 1/8 + 1/16 + \ldots$$

- This means that we are forever adding smaller and smaller fractions without ever stopping our addition.
- So is $1/2 + 1/4 + 1/8 + 1/16 + \ldots = \infty$?

Examples

2) *Infinity and the infinitely small*

- Also, consider the following example:

$$1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \ldots$$

What is the answer to this? Is it

– $(1 - 1) + (1 - 1) + (1 - 1) + \ldots = 0$,

or

– $1 + (- 1 + 1) + (- 1 + 1) + (- 1 + 1) + \ldots = 1$

Examples

2) *Infinity and the infinitely small*

or if we let $S = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \ldots$ then

$$S = \quad +1 - 1 + 1 - 1 + 1 - 1 + \cdots$$

$$S = 1 \quad -1 + 1 - 1 + 1 - 1 + \cdots$$

$$2S = 1$$

implying $S = ½$. Three answers for the same addition!!

Examples

2) *Infinity and the infinitely small*

- So the Greeks decided to banish all maths involving infinities and the infinitely small.
- Such was the opinion of experts.

Absolutism and relativism

Absolutism

- Some object or phenomenon is always true, independently of historical, social or cultural context. For example,
  - Gravity: objects always fall to Earth;
  - Aerodynamics: aerodynamic lift allows planes to fly;
  - Buoyancy: objects heavier than water are able to float;

Absolutism and relativism

Absolutism

- The mathematical description may change over time (Newton’s formulation of gravity as force, then Einstein’s formulation of gravity as curved spacetime) …
- … but the phenomena still behave as they have always behaved.

Absolutism and relativism

Absolutism

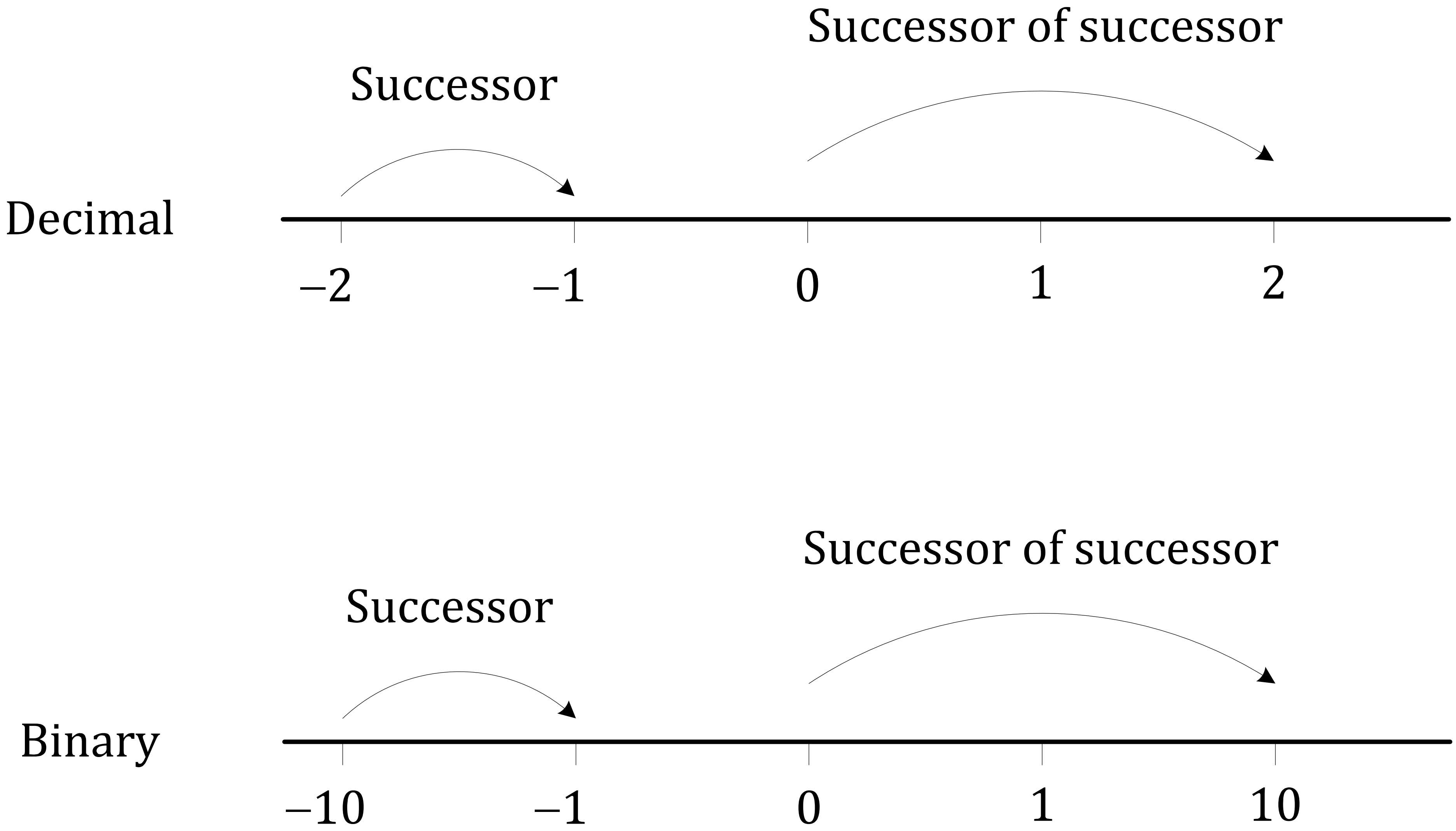
- In reference to exercise 1 some people may say arithmetic is relative.
- I do not agree. It is true that the answers depend, or are contingent, on the number base you use (binary, decimal, hexadecimal, etc.).

Absolutism and relativism

Absolutism

- There aren't different mathematical truths about arithmetic.
- There are simply different *representations* about an underlying truth of arithmetic.
- The fundamental truth about arithmetic can be described as "successor to"

Absolutism and relativism



Absolutism and relativism

Absolutism

- So mathematics may be contingent: improved mathematical formulation as time progresses …
- … but mathematics references an absolute truth: arithmetic, gravity, aerodynamics, hydrostatic, chemical process, etc.

Absolutism and relativism

Relativism

- *Definition 1 (vs 1): Co-variant relativism*

  An object or phenomenon is dependent on the context or domain of applicability.

- *Example*

  In exercise 1) above the results (the objects) of the arithmetic are relative to the number base (context or domain of applicability).

Absolutism and relativism

Relativism

- *Definition 1 (vs 2): Conceptual relativism*

  Certain concepts are true only with respect to a given frame of reference.

- *Example*

  Two lines that are parallel (the concept) are always parallel is relative to Euclidean geometry (frame of reference), since in other geometries (hyperbolic, elliptical, etc) this is not true.

Absolutism and relativism

Relativism

- See notes for a third definition.

Absolutism and relativism

Problems with relativism

- Einstein’s 1915 theory of general relativity said the universe was either expanding or collapsing
- The scientific belief of the time was that the universe was static.
- As a result, Einstein included an extra term in his equations to make these produce a static universe.

Absolutism and relativism

Problems with relativism

- In 1929 Edwin Hubble collected data which showed the universe was expanding.
- Einstein called his insertion of this extra term the biggest mistake of his life.

Absolutism and relativism

Problems with relativism

- Mathematics and science are not democracies. It is neither the most common or most popular conclusion that becomes accepted.
- Instead, it is the conclusion that stands up to the test of evidence over time.

Exercises

Commentary

- Read the commentary, p11-14 of the notes.

Exercises

- See notes.